Multiresolution Visualization of Digital Earth Data via Hexagonal Box-Spline Wavelets

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Abstract

Multiresolution analysis is an important tool for exploring large-scale data sets. Such analysis provides facilities to visualize data at different levels of detail while providing the advantages of efficient data compression and transmission. In this work, an approach is presented to apply multiresolution analysis to digital Earth data where each resolution describes data at a specific level of detail. Geospatial data at a fine level is taken as the input and a hierarchy of approximation and detail coefficients is built by applying a hexagonal discrete wavelet transform. Multiresolution filters are designed for hexagonal cells based on the three directional linear box spline which is natively supported by modern GPUs.

Keywords: Multiresolution analysis, digital earth data, linear box-spline, hexagonal grid.

1 Introduction

Digital Earth, or virtual Globe, has become an important subject in the field of Climatology. Climate research relies on large amounts of geospatial data obtained from various kinds of acquisition or simulation processes, and requires an appropriate framework to represent this data in a structured manner. In most cases, such frameworks deal with discretizing the Earth's surface into different geometric objects and the governing equations of state for the simulation are solved on the resulting grid. These cells represent areas that contain geospatial information related to the point of interest.

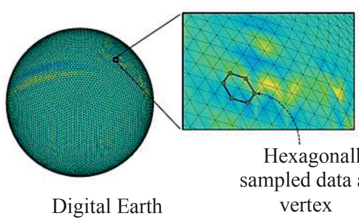

Figure 1: Data sampled on a hexagonal grid. The Voronoi cell at each vertex is a hexagon.

Owing to the vast amount of data, a level of detail (LoD) scheme is typically employed in order to facilitate data management and visualization. Such schemes allow a user to quickly browse data at a lower LoD while a higher LoD is presented on demand as the user zooms in on an area of interest. A straightforward approach to obtain different LoDs is to simply downsample the original data and store it at several coarse resolutions. However, this results in a drastic loss in quality between the levels and a lot of data duplication.

To remedy these issues, we propose to apply multiresolution analysis (MRA) to digital Earth data. Mathematically, an MRA is a nested set of linear function spaces $V^0 \subset V^1 \subset \cdots \subset V^n$, with the resolution of functions in V^j increasing with j [1]. Wavelets, a key ingredient in MRA, represent data at multiple scales and can recover data at a finer resolution from a coarser resolution via the addition of details. To further elaborate, multiresolution encapsulates two processes: decomposition and reconstruction. Decomposition is the process of downsampling some fine data F into a coarse approximation $C = [c_0,\ c_1, \ldots]^T$ and detail vectors or coefficients $D = [d_0,\ d_1, \ldots]^T$, known as the wavelet or detail coefficients. The details allow the original fine data to be perfectly reconstructed. Formally,

$$C = AF,$$
$$D = BF,$$

where A and B are matrices known as *decomposition filters*. Reconstruction is the process of increasing the resolution of coarse data C to produce fine data $F = [f_0,\ f_1, \ldots]^T$. When the multiresolution is lossless, the original fine data F can be perfectly reproduced in the reconstruction process. Formally,

$$F = PC + QD,$$

where P and Q are matrices known as *reconstruction filters*. Together, P, Q, A and B are known as the *multiresolution filters*. Variations in these matrices produce different multiresolution schemes.

In this paper, a MRA is designed for hexagonally sampled (see Figure-1) digital Earth data based on the Fast Wavelet Transform [2]. Usually, wavelets that operate on rectangular cells are more popular owing to their simplicity. The computational geosciences community has historically used latitude-longitude (regular structured) grids. But due to reasons of improved scalability, and better handling of the singularities that result at the poles with latitude-longitude grids, the computational geosciences community is moving toward less regular discretizations. A major contribution of this work is to explore, in the context of digital Earth, wavelets that operates on hexagonal cells. The next section of this paper presents the data structure used to work with digital Earth followed by a high level overview of the proposed multiresolution scheme. Finally, it concludes with some preliminary results.

2 Methodology

2.1 Atlas of Connectivity Maps

An efficient data structure is necessary when dealing with digital Earth information. In this work, we employ Atlas of Connectivity Maps (ACM) [3], which maps vertex connectivity information to separate two-dimensional arrays. Many digital Earth models are obtained by refining an icosahedron. ACM splits the icosahedron into a set of diamonds. Vertex information on the diamonds is stored in individual 2D arrays. Thus, the connectivity information of the entire digital Earth model is mapped to ten different arrays. This type of data structure provides simple and efficient ways to

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traverse the vertices and retrieve neighbourhood information. Fig-2 illustrates the idea of ACM applied to digital Earth.

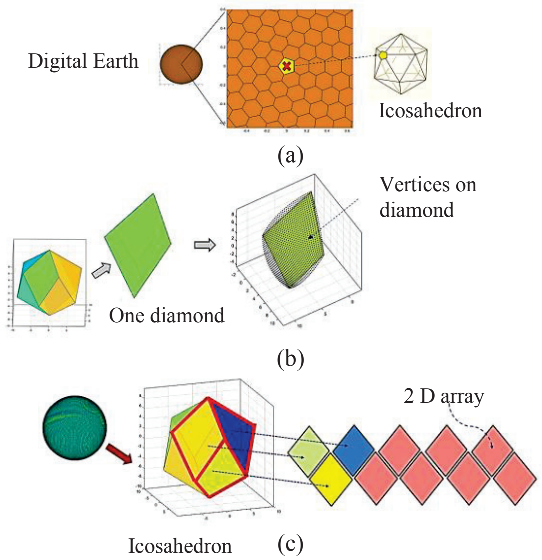

Figure 2: Atlas of Connectivity Maps on Digital Earth. (a) Icosahedron based on pentagonal vertices. (b) Vertices on the region of a single diamond. (c) Concept of storing connectivity information of all the diamonds in ten arrays.

2.2 Multiresolution Scheme

Our proposed multiresolution scheme is based on [2] and [4]. In particular, we employ a hexagonally supported three-directional linear box spline [5] as the scaling function in our MRA. Data reconstruction with this box spline is tantamount to barycentric interpolation within triangular cells which is natively supported on GPUs. In this multiresolution scheme, the decomposition and reconstruction steps are efficiently performed via discrete convolution operations. The decomposition step is performed as follows:

$$C = (F * \mathring{w}_0)_{\downarrow 4}\ ,$$

$$D_i = (F * \mathring{w}_i)_{\downarrow 4}\ , \quad \text{where } i = 1,2,3.$$

Here, F is the fine resolution, C and D_i are coarse-scale coefficients and details respectively, and "$\downarrow 4$" indicates a dyadic downsampling operation in each direction. The weight filters $\mathring{w}_i$ are obtained by exploiting the fact that the detail spaces constitute the orthogonal complement of the coarse-scale space with respect to fine level.

The reconstruction step follows a similar method:

$$F = (C)_{\uparrow 4} * w_0 + \sum_{i=1}^{3}((D_i)_{\uparrow 4} * w_i)\ ,$$

where F is the fine level which is reconstructed perfectly from coarse level coefficients C and details D_i. The symbol "$\uparrow 4$" indicates a dyadic upsampling operation. The filters w_i are obtained from $\mathring{w}_i$ by exploiting the fact that w_i and $\mathring{w}_i$ form a biorthogonal system. Figure-3 (a) illustrates the schematic diagram of the multiresolution process.

This multiresolution scheme is applied separately to each diamond. Coherence between the diamonds can be achieved by imposing appropriate boundary conditions for the convolution operation.

3 Preliminary Results

We have tested our proposed scheme on ICON (ICOsahedral Non-hydrostatic) datasets. The ICON format is a joint venture of the German Weather Service (DWD) and the Max-Planck-Institute for Meteorology (MPI-M), and is used for numerical climate simulations. The ACM data structure is applied to this dataset and the connectivity information of ICON is stored in 2D arrays. Our multiresolution scheme is also applied to ICON to observe the results of the decomposition and reconstruction steps. Figure-3 (b) shows result of our implementation. While the decomposition step is working well, the results show that the reconstructed output is not satisfactory. Future work includes investigating the source of the error and the impact of boundary conditions.

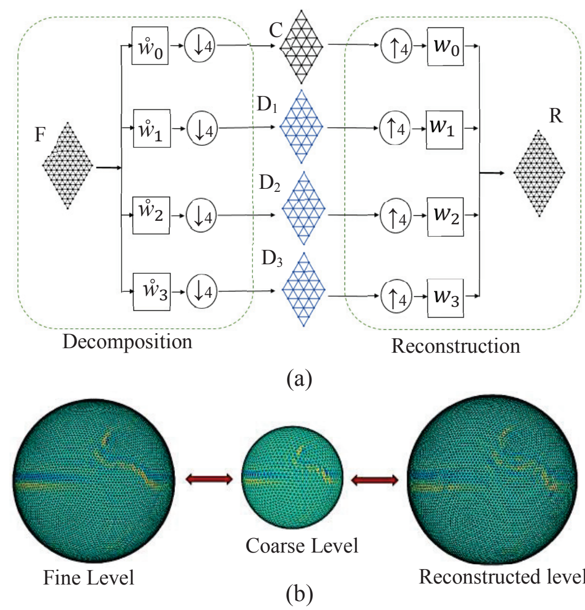

Figure 3: (a) Shows schematic diagram of the proposed multiresolution technique where F is the fine level, C is coarse level, D_1, D_2, D_3 are details and R is the reconstructed level. (b) Shows the multiresolution analysis of the entire digital Earth model.

4 Conclusion

This paper presents a work in progress where hexagonal wavelets are used for the multiresolution visualization of digital Earth. We believe that successful completion of this work will provide an efficient multiresolution scheme that can be applied to hexagonally sampled data for LoD visualization as well as data compression.

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